

# Quantum entanglement and quantum nonlocality for $N$ -photon entangled states

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Quantum entanglement and quantum nonlocality of  $N$ -photon entangled states  $|\psi_{Nm}\rangle = \mathcal{N}_m[\cos\gamma|N-m\rangle_1|m\rangle_2 + e^{i\theta_m}\sin\gamma|m\rangle_1|N-m\rangle_2]$  and their superpositions are studied. We indicate that the relative phase  $\theta_m$  affects quantum nonlocality but not quantum entanglement for the state  $|\psi_{Nm}\rangle$ . We show that quantum nonlocality can be controlled and manipulated by adjusting the state parameters of  $|\psi_{Nm}\rangle$ , superposition coefficients, and the azimuthal angles of the Bell operator. We also show that the violation of the Bell inequality can reach its maximal value under certain conditions. It is found that quantum superpositions based on  $|\psi_{Nm}\rangle$  can increase the amount of entanglement, and give more ways to reach the maximal violation of the Bell inequality. PACS number(s): 03.65.Ud, 03.67.-a

## I. INTRODUCTION

Quantum entanglement and quantum nonlocality are two striking aspects of quantum mechanics. They are introduced by Einstein, Podolsky, and Rosen (EPR) in their famous paper [1]. The relationship between them has been paid much attention. They play an essential role in the modern understanding of quantum phenomena, and quantum information transmission and processing. Bell proposed a remarkable inequality imposed by a local hidden variable theory [2], which enables a quantitative test on quantum nonlocality. The quantum nonlocality test can be performed on an entangled system composed of two coherent systems. This entangled system can be used as a quantum entangled channel for quantum information transfer. Numerous theoretical studies and experimental demonstrations have been carried out to understand nonlocal properties of quantum states. Various versions of Bell's inequality [3, 4] are proposed. Gisin and Peres found pairs of observable whose correlations violate Bell's inequality for a discrete  $N$ -dimensional entangled state [5]. Banaszek and Wódkiewicz studied Bell's inequality for continuous-variable states in terms of Wigner representation in phase space based upon parity measurement and displacement operation. Recently, Chen *et al.* studied Bell's inequality of continuous-variable states [6] using their newly defined pseudospin Bell operators, and they showed that the EPR state can maximally violate Bell's inequality in their framework.

Recently,  $N$ -photon entangled states and their superposition states are paid much attention. They are widely used to realize quantum lithography [7, 8, 9], super-resolving phase measurements [10], and quantum teleportation [11]. In this letter, we study quantum entanglement and quantum nonlocality of the following  $N$ -photon entangled states and their superpositions

$$|\psi_{Nm}\rangle = \mathcal{N}_m[\cos\gamma|N-m\rangle_1|m\rangle_2 + e^{i\theta_m}\sin\gamma|m\rangle_1|N-m\rangle_2], \quad (1)$$

where  $m$  takes its values from zero to  $N$ , the normalization factor is given by  $\mathcal{N}_m^{-2} = 1 + \cos\theta_m \sin 2\gamma \delta_{N,2m}$  with  $\gamma$  and  $\theta_m$  being an entanglement angle and a relative phase, respectively. We will calculate the von Neumann entropy and study the Bell's inequality for quantum states  $|\psi_{Nm}\rangle$  and their superposition states. We will show that quantum superpositions for the two modes may increase the amount of entanglement, and  $N$ -photon entangled states can maximally violate Bell's inequality. This letter is organized as follows. In Sec. II, we study quantum entanglement of  $N$ -photon entangled states and their superposition states through analyzing their von Neumann entropy. Quantum nonlocality of  $N$ -photon entangled states and their superposition states is investigated through discussing the violation of the Bell's inequality in the pseudospin Bell-operator formalism developed by Chen *et al.* [6] in Sec. III. The last section is devoted to summary and conclusion.

## II. QUANTUM ENTANGLEMENT FOR $N$ -PHOTON ENTANGLED STATES

In this section, we study properties of quantum entanglement of the  $N$ -photon entangled states given by Eq. (1). In order to this, we consider the nontrivial case of  $N \neq 2m$ , in which the normalization constant  $\mathcal{N}_m = 1$ . The degree of the entanglement can be described by the von Neumann entropy defined by

$$E(\rho_1) = -\text{Tr}_1(\rho_1 \log \rho_1), \quad (2)$$

where  $\rho_1$  is the reduced density operator of the first mode.

For the  $N$ -photon entangled states defined by Eq. (1) we find the von Neumann entropy to be

$$E_1 = -\cos^2\gamma \log \cos^2\gamma - \sin^2\gamma \log \sin^2\gamma, \quad (3)$$

which indicates that quantum entanglement of the  $N$ -photon entangled states given by Eq. (1) is independent of the superposition phases  $\theta_m$  and the total photon number of the two modes  $N$ , and changes periodically with respect to the entanglement angle  $\gamma$ . In particular, the amount of entanglement reaches the maximal value of

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$E_1 = 1$  when the entanglement angle takes values by  $\gamma = k\pi + \pi/4$  with  $k$  being an integer. In Fig. 1 we plot the change of the von Neumann entropy with respect to the entanglement angle.

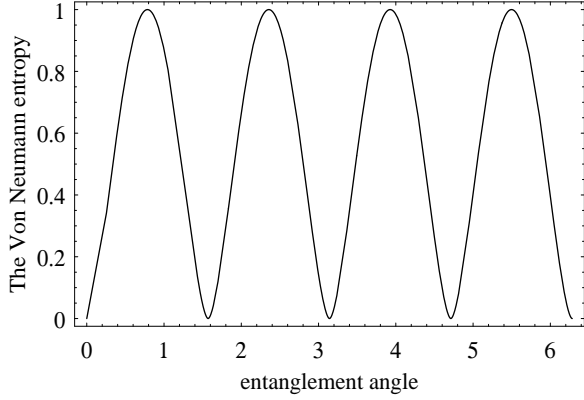


FIG. 1: The von Neumann entropy of the  $N$ -photon entangled state is plotted against the entanglement angle  $\gamma$ .

We then study quantum entanglement of superposition states based on the  $N$ -photon entangled states given by Eq. (1). Firstly, we consider a two-state superposition state defined by

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|\psi_{30}\rangle + |\psi_{31}\rangle), \quad (4)$$

which leads to the von Neumann entropy

$$E_2 = 1 - \cos^2 \gamma \log \cos^2 \gamma - \sin^2 \gamma \log \sin^2 \gamma, \quad (5)$$

which implies that the amount of entanglement of the two-component superposition state only depends on the entanglement angle  $\gamma$  of its basis. Especially, comparing Eq. (5) with (5) we find that the difference of the amount of entanglement between the two-component superposition state (4) and the basis state (1) is a positive constant, i.e.,  $E_2 - E_1 = 1$ . This indicates that the entanglement amount of the superposition state is always larger than that of the basis state for an arbitrary entanglement angle  $\gamma$ . In other words, starting with basis states defined by (1) one can construct quantum superposition states with larger amount of entanglement than that of the basis states. Hence, we may conclude that quantum superpositions for the two modes may increase the amount of entanglement.

In order to further demonstrate the above idea of quantum superpositions increasing quantum entanglement, in what follows we take into account a multi-component quantum superposition state consisting of  $N$  bases with a fixed photon number  $N$ , but with different distributions  $m$ ,

$$|\psi_N\rangle = A \sum_{m=0}^N \alpha_m |\psi_{Nm}\rangle, \quad (6)$$

where  $A$  is a normalization factor. It is straightforward to express the  $N$ -component superposition state as the following number-sum Bell state,

$$|\Psi_N\rangle = \sum_{m=0}^N d_m |N-m\rangle_1 |m\rangle_2, \quad (7)$$

which is the eigenstate of the number-sum Bell operators  $\hat{N} = \hat{N}_1 + \hat{N}_2$  with  $\hat{N}_i$  being the number operators of the respective modes. The coefficients in Eq. (7) are given by

$$d_m = A(\alpha_m \mathcal{N}_m \cos \gamma + \alpha_{N-m} \mathcal{N}_{N-m} e^{i\theta_{N-m}} \sin \gamma), \quad (8)$$

where the normalization of the superposition state (8) implies that  $d_m$  satisfy the condition  $\sum_{m=0}^N |d_m|^2 = 1$ .

In order to calculate the von Neumann entropy of the superposition state (7), we need the reduced density operator

$$\rho_1 = \sum_{m=0}^N |d_m|^2 |N-m\rangle_1 \langle N-m|, \quad (9)$$

which leads to the following von Neumann entropy

$$E_N = - \sum_{m=0}^N |d_m|^2 \log |d_m|^2. \quad (10)$$

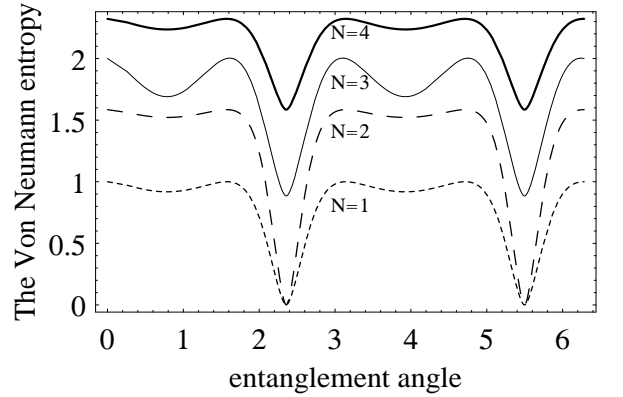


FIG. 2: The von Neumann entropy of multi-component superposition states based on  $|\psi_{Nm}\rangle$  is plotted against the entanglement angle  $\gamma$  for  $N = 1, 2, 3$ , and  $4$ , respectively.

For the sake of simplicity and without the loss of generality, we consider the situation of equal-weight superposition in which we choose the superposition coefficients  $\alpha_m = 1/\sqrt{N+1}$  and the relative phases  $\theta_m = 2\pi m/N$ . In this case, we have

$$|d_m|^2 = A^2 |\mathcal{N}_m|^2 \{1 + \cos[2\pi(N-m)/N] \sin 2\gamma\}, \quad (11)$$

where the normalization constant is given by

$$A^{-2} = \sum_{m=0}^N |\mathcal{N}_m|^2 \{1 + \cos[2\pi(N-m)/N] \sin 2\gamma\}. \quad (12)$$

Then the von Neumann entropy of the superposition state can be directly obtained through substituting (11) and (12) into (10). From Eqs. (10)-(12) we can find the maximal value of the von Neumann entropy of the superposition state to be  $E_{N,max} = \log_2(N+1)$  when  $\gamma = k\pi/2$  with  $k$  being an arbitrary integer. This implies that the maximal amount of entanglement of the  $N$ -component superposition state only depends on the total photon number  $N$ , and increases with increasing the total photon number  $N$ . In order to clearly see the influence of the number of components  $N$  and the entanglement angle  $\gamma$ , in Fig. 2 we plot the von Neumann entropy of the superposition states when  $N = 1, 2, 3$ , and 4, respectively. From Fig. 2 we can see that the entanglement amount increases with increasing the number of components, i.e., the photon number  $N$ , and changes periodically with respect to the entanglement angle  $\gamma$ .

### III. QUANTUM NONLOCALITY FOR $N$ -PHOTON ENTANGLED STATES

In this section, we study quantum nonlocality of  $N$ -photon entangled states and their superposition states through discussing the violation of the Bell's inequality in the pseudospin Bell-operator formalism developed by Chen and coworkers [6]. Let us begin with a brief review of the pseudospin-operator formalism [6]. For a single-mode boson field, the pseudospin operators can be defined in terms of project operators in a Fock space in the following form

$$\begin{aligned} S_z &= \sum_{n=0}^{\infty} [|2n+1\rangle\langle 2n+1| - |2n\rangle\langle 2n|], \\ S_- &= \sum_{n=0}^{\infty} [|2n\rangle\langle 2n+1|] = (S_{j+})^\dagger, \end{aligned} \quad (13)$$

where  $|n\rangle$  are the usual Fock states of the boson mode. The operator  $S_z = -(-1)^{\hat{N}}$  with  $\hat{N}$  being the number operator and  $(-1)^{\hat{N}}$  being the parity operator,  $S_+$  and  $S_-$  being the ‘‘parity-flip’’ operators. They satisfy the commutation relations of the  $su(2)$  Lie algebra

$$[S_z, S_\pm] = \pm 2S_\pm, \quad [S_+, S_-] = S_z. \quad (14)$$

For an arbitrary vector living on the surface of a unit sphere  $\vec{a} = (\sin \theta_a \cos \varphi_a, \sin \theta_a \sin \varphi_a, \cos \theta_a)$ , we have the following dot product

$$\vec{a} \cdot \vec{S} = S_z \cos \theta_a + \sin \theta_a (e^{i\varphi_a} S_- + e^{-i\varphi_a} S_+). \quad (15)$$

Then for a two-mode boson field, the Bell operator due to Clauser, Horne, Shimony, and Holt (CHSH) [?] can be defined by

$$\begin{aligned} B &= (\vec{a} \cdot \vec{S}_1) \otimes (\vec{b} \cdot \vec{S}_2) + (\vec{a} \cdot \vec{S}_1) \otimes (\vec{b}' \cdot \vec{S}_2) \\ &\quad + (\vec{a}' \cdot \vec{S}_1) \otimes (\vec{b} \cdot \vec{S}_2) - (\vec{a}' \cdot \vec{S}_1) \otimes (\vec{b}' \cdot \vec{S}_2), \end{aligned} \quad (16)$$

where  $\vec{a}', \vec{b}$ , and  $\vec{b}'$  are three unit vectors similarly defined as  $\vec{a}, \vec{S}_1$  and  $\vec{S}_2$  are defined as in Eq. (13).

As well known, local hidden variable theories impose the Bell-CHSH inequality  $|\langle B \rangle| \leq 2$  where  $\langle B \rangle$  is the mean value of the Bell operator with respect to a given quantum state. However, in the quantum theory it is found that  $|\langle B \rangle| \leq 2\sqrt{2}$ , which implies that the Bell-CHSH inequality is violated. In particular, when  $|\langle B \rangle| = 2\sqrt{2}$  for a given quantum state, we say that the Bell-CHSH inequality is maximally violated by the quantum state.

Quantum nonlocality of a quantum state can be described by the violation of the Bell-CHSH inequality. The expectation value of the Bell operator with respect to a quantum state  $|\psi\rangle$  can be expressed in terms of the correlation functions as

$$\begin{aligned} \langle B \rangle &= E(\theta_a, \theta_b) + E(\theta_a, \theta_{b'}) + E(\theta_{a'}, \theta_b) \\ &\quad - E(\theta_{a'}, \theta_{b'}), \end{aligned} \quad (17)$$

where the correlation functions are defined by

$$E(\theta_a, \theta_b) = \langle \psi | S_{\theta_a}^{(1)} \otimes S_{\theta_b}^{(2)} | \psi \rangle, \quad (18)$$

with

$$S_{\theta_a}^{(j)} = S_{jz} \cos \theta_a + S_{jx} \sin \theta_a. \quad (19)$$

We now investigate quantum nonlocality of the  $N$ -photon entangled state given by Eq. (1). For this multi-photon entangled state we find the correlation function to be

$$\begin{aligned} E_{Nm}(\theta_a, \theta_b) &= \mathcal{N}_m^2 \{ [(-1)^N + K(\theta_m, \gamma) \delta_{N,2m}] \\ &\quad \times \cos \theta_a \cos \theta_b + \delta_{N,2m \pm 1} \\ &\quad \times K(\theta_m, \gamma) \sin \theta_a \sin \theta_b \}, \end{aligned} \quad (20)$$

where we have introduced the following effective state parameter

$$K(\theta_m, \gamma) = \cos \theta_m \sin 2\gamma, \quad (21)$$

which describes the effect of the basis state defined by (1) on the correlation functions.

Making use of the correlation function (20), from Eq. (17) we can get the expectation value of the Bell operator for arbitrary values of all azimuthal angles  $\theta_a, \theta_b, \theta_{a'}$  and  $\theta_{b'}$ . As a concrete example, we consider the situation of  $\theta_a = 0, \theta_{a'} = \pi/2$  and  $\theta_b = -\theta_{b'}$ . In this case, from Eqs. (17), (20) and (21) we can obtain the expectation value of the Bell operator given by

$$\begin{aligned} \langle B \rangle &= 2\mathcal{N}_m^2 \{ [(-1)^N + K(\theta_m, \gamma) \delta_{N,2m}] \cos \theta_b \\ &\quad + \delta_{N,2m \pm 1} K(\theta_m, \gamma) \sin \theta_b \}. \end{aligned} \quad (22)$$

In order to observe quantum nonlocality of the  $N$ -photon entangled state (1), we consider two different cases of  $N = 2m$  and  $N = 2m \pm 1$ , respectively. When

$N = 2m$ , the quantum state given by Eq. (1) is disentangled, and reduces to

$$|\psi_{Nm}\rangle = \mathcal{N}_m(\cos \gamma + e^{i\theta_m} \sin \gamma)|m\rangle_1|m\rangle_2. \quad (23)$$

Making use of Eqs. (17), (18) and (20), we obtain the expectation value of the Bell operator with respect to the disentangled state (23)  $\langle B \rangle = 2 \cos \theta_b$ , which means that  $|\langle B \rangle| \leq 2$ . Hence, the unentangled state (23) cannot produce a violation of Bell's inequality.

On the other hand, when  $N = 2m \pm 1$ , the expectation value of the Bell operator with respect to the state (1) is given by

$$\langle B \rangle = 2[K(\theta_m, \gamma) \sin \theta_b - \cos \theta_b], \quad (24)$$

which indicates that for a fixed entanglement angle  $\gamma$  and a fixed relative phase  $\theta_m$ , when  $\theta_b = -\tan^{-1} K$ , the expectation value of the Bell operator  $\langle B \rangle$  reaches its maximum given by

$$\langle B \rangle_{max} = 2\sqrt{1 + K^2(\theta_m, \gamma)}, \quad (25)$$

which implies that  $\langle B \rangle_{max} \geq 2$  due to  $|K(\theta_m, \gamma)| \leq 1$ . Thus, the  $N$ -photon entangled state always violates the Bell's inequality if  $K(\theta_m, \gamma) \neq 0$ . From Eqs. (21) and (24) we can see that the degree of violation of the Bell's inequalities depends upon both the entangling angle  $\gamma$  and the relative phase  $\theta_m$ , and it changes periodically with respect to both  $\gamma$  and  $\theta_m$ . In particular, we note that the relative phase  $\theta_m$  seriously affects the mean value of the Bell operator although it does not affect quantum entanglement of the quantum state given by (1). It is easy to find that when  $|\psi_{Nm}\rangle$  reaches maximal entanglement i.e.,  $\gamma = \pi/4$ , and  $\theta_m = 0$ , we have  $K(\theta_m, \gamma) = 1$ . Under these conditions, we can reach the maximal violation of Bell's inequality with  $\langle B \rangle_{max} = 2\sqrt{2}$ .

In Fig. 3 we plot the degree of the violation of the Bell's inequality for  $N$ -photon entangled state against  $\gamma$  and  $\theta_m$ . From Fig. 3 we can see that the mean value of the Bell operator with respect to the multi-photon entangled state (1) changes periodically with both of the entanglement angle and the relative phase, and the Bell's inequality is always violated for arbitrary values of the entanglement angle and the relative phase except  $\gamma = k\pi/2$  and  $\theta = (2k+1)\pi/2$  with  $k$  is an integer.

Finally, we consider quantum nonlocality of quantum superposition states which is formed using the multi-photon entangled state (1). In order to calculate the expectation value of the Bell operator with respect to the superposition states based on entangled states given by (1), we need the following correlation function

$$E_{mm'} = \langle \psi_{Nm} | S_{\theta_a}^{(1)} \otimes S_{\theta_b}^{(2)} | \psi_{Nm'} \rangle, \quad (26)$$

which is given by the following expression

$$E_{mm'} = \mathcal{N}_m \mathcal{N}_{m'}^* \cos \theta_a \cos \theta_b \times \{(-1)^N \delta_{m,m'} \delta_{N,m+m'} \cos \gamma \sin \gamma$$

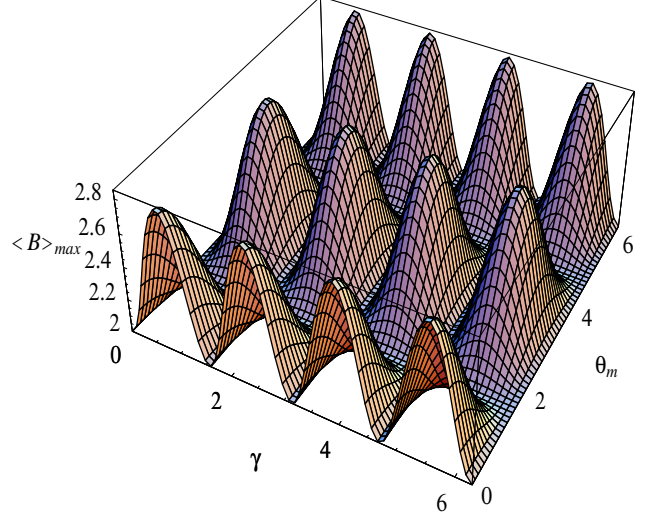


FIG. 3: The degree of the maximal violation of the Bell's inequality for  $N$ -photon entangled state  $\langle B \rangle_{max}$  is plotted against the entanglement angle  $\gamma$  and the relative phase  $\theta_m$ .

$$\begin{aligned} & \times [\cos^2 \gamma + e^{i(\theta_{m'} - \theta_m)} \sin^2 \gamma] \\ & + (-1)^{m+m'} (e^{i\theta_{m'}} + e^{-i\theta_m}) \} \\ & + \mathcal{N}_m \mathcal{N}_{m'}^* \sin \theta_a \sin \theta_b \\ & \times \{ [\cos^2 \gamma + e^{i(\theta_{m'} - \theta_m)} \sin^2 \gamma] \delta_{m,m' \pm 1} \\ & + \cos \gamma \sin \gamma (e^{i\theta_{m'}} + e^{-i\theta_m}) \delta_{N,m+m' \pm 1} \} \end{aligned} \quad (27)$$

As a simple example of analyzing quantum nonlocality of superposition states, we consider the following two-component superposition state,

$$|\psi_3\rangle = C(\alpha_0 |\psi_{30}\rangle + \alpha_1 |\psi_{31}\rangle), \quad (28)$$

where  $C$  is the normalization constant. Making use of Eq. (27), we can obtain the expectation value of the Bell operator for this superposition state

$$\begin{aligned} \langle B \rangle &= -2 \cos \theta_b + 4 \sin \theta_b (|\alpha_0|^2 + |\alpha_1|^2)^{-1} \\ & \times \text{Re}[\alpha_0^* \alpha_1 (\cos^2 \gamma + e^{i(\theta_1 - \theta_0)} \sin^2 \gamma)], \end{aligned} \quad (29)$$

which indicates that  $\langle B \rangle$  depends upon not only the superposition coefficients  $\alpha_0, \alpha_1$ , the azimuthal angles  $\theta_a, \theta_b$ , but also upon the state parameters of the basis states given by Eq. (1),  $\theta_0, \theta_1$  and  $\gamma$ .

In order to observe the maximal violation of the Bell inequality, we can choose  $\alpha_0 = \alpha_1 = 1$ , the Bell function  $\langle B \rangle$  given by Eq. (29) becomes

$$\begin{aligned} \langle B \rangle &= 2 \sin \theta_b [\cos^2 \gamma + \cos(\theta_1 - \theta_0) \sin^2 \gamma] \\ & - 2 \cos \theta_b. \end{aligned} \quad (30)$$

Making use of Eq. (30), we can show that the maximal violation of the Bell inequality can be reached by

controlling the azimuthal angle  $\theta_b$ , the state parameters of the basis states given by Eq. (1),  $\theta_0, \theta_1$  and  $\gamma$ . In fact, when  $\theta_1 - \theta_0 = 2k\pi$  with  $k$  being an arbitrary integer, we can arrive at the following Bell function

$$\langle B \rangle = 2(\sin \theta_b - \cos \theta_b) \quad (31)$$

which means that the values of the Bell function of  $\langle B \rangle$  are independent of the entangling parameter of the basis state defined in (1), and we can reach the maximal violation of the Bell inequality  $\langle B \rangle_{max} = 2\sqrt{2}$  when  $\theta_b = -\pi/4$ .

On the other hand, when the phase difference takes  $\theta_1 - \theta_0 = (2k+1)\pi$  with  $k$  being an arbitrary integer, the Bell function given by Eq. (30) becomes

$$\langle B \rangle = 2[\sin \theta_b \cos(2\gamma) - \cos \theta_b], \quad (32)$$

from which we can see that when the entanglement angle satisfies  $\gamma = k\pi$  with  $k$  being an arbitrary integer, Eq. (32) becomes Eq. (31), hence the maximal violation of the Bell inequality is reached with  $\theta_b = -\pi/4$ . However, when the entanglement angle satisfies  $\gamma = (2k+1)\pi/2$  with  $k$  being an arbitrary integer, the Bell function (32) reduces to

$$\langle B \rangle = -2(\sin \theta_b + \cos \theta_b), \quad (33)$$

which implies that the violation of the Bell inequality reaches its maximal value  $\langle B \rangle_{max} = 2\sqrt{2}$  when  $\theta_b = \pi/4$ . Therefore, we can conclude that superposition states based on entangled states (1) can provide us with more ways of reaching the maximal violation of the Bell's inequality.

#### IV. CONCLUDING REMARKS

In summary, we have studied quantum entanglement and quantum nonlocality of  $N$ -photon entangled states for the two modes defined in Eq. (1) and their superpositions through investigating the von Neumann entropy and the violation of the Bell's inequality in the pseudospin Bell-operator formalism. For the multi-photon entangled states defined in Eq. (1) we have indicated

that the von Neumann entropy is independent of the relative phase  $\theta_m$ . Hence, quantum entanglement of these states only depends on the entanglement angle. However, quantum nonlocality of these quantum states exhibits different dependence upon the state parameters. We have shown that both of the entanglement angle and the relative phase seriously affect the violation of the Bell's inequality. And we have indicated that under certain conditions the maximal violation of the Bell's inequality can be reached.

It is worthwhile to mention that multi-component superposition states made from  $N$ -photon entangled states for the two modes defined in Eq. (1) exhibit some interesting characteristics on their quantum entanglement and quantum nonlocality. Firstly, we have found that these multi-component superposition states have larger amount of entanglement than that of the basis states defined by (1). Hence, quantum superpositions for the two modes can increase the amount of entanglement. This indicates the possibility of obtaining entangled states with a larger amount of entanglement starting from entangled states with a smaller amount of entanglement. Secondly, we have found that for these quantum superposition states there are more ways to make corresponding Bell's inequality reach the maximal violation, and revealed that quantum nonlocality can be controlled and manipulated by adjusting the state parameters of  $|\psi_{Nm}\rangle$ , superposition coefficients, and the azimuthal angles of the Bell operator. We hope that these results obtained in present paper would find their applications in quantum information processing [12, 14, 15, 16] and the test of quantum nonlocality [17, 18].

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